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201EE012

Ph.D./M.S.(Engg.) by Research Degree Examination, Aug./Sep. 2023
Analysis of Linear Systems

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive the state model for the electrical network shown in Fig.Q1(a) consider voltage across R_2 and current through R_2 as output variables.

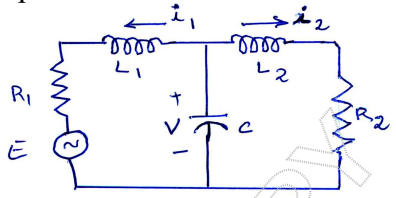

Fig.Q1(a) **(08 Marks)**

b. Obtain state model in controllable and observable canonical form for the following system

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$ **(08 Marks)**

c. Draw parallel realization and resulting state model for the system described by

$$\dddot{y}(t) + 6\ddot{y}(t) + 11\dot{y}(t) + 6y(t) = u(t)$$ **(04 Marks)**

OR

2 a. Show that the input output behavior of state model is invariant under linear transformation. **(06 Marks)**

b. Determine eigen values, eigen vectors and diagonalize the state model.

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 & 0 \\ 1 & -3 & 0 \\ -1 & -2 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0 \quad 0]x(t)$$ **(08 Marks)**

c. Represent the system shown in Fig.Q2(c) using state model.

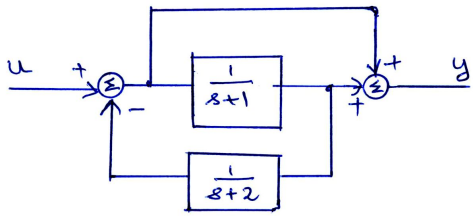

Fig.Q2(c) **(06 Marks)**

Module-2

3 a. Derive transfer function of a zero-order hold. **(06 Marks)**

b. Obtain pulse transfer function of the system shown in Fig.Q3(b).

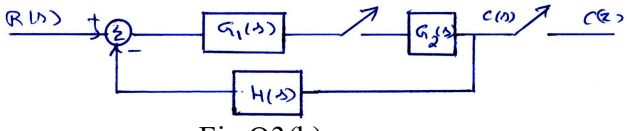

Fig.Q3(b) **(07 Marks)**

c. Consider a function $x(s) = \frac{1-e^{-Ts}}{s} G(s)$, prove that $x(z) = (1-z^{-1}) z\left\{\frac{G(s)}{s}\right\}$ **(07 Marks)**

OR

4 a. With illustration explain quantizing, quantization error and round off error. Draw the probability distribution of quantizing error. **(06 Marks)**

b. Obtain pulse transfer function of the system shown in Fig.Q4(b).

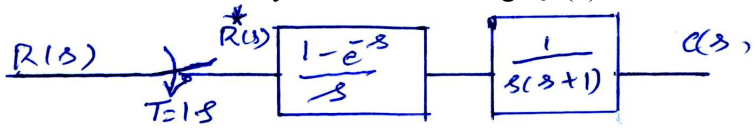

Fig.Q4(b) **(08 Marks)**

c. The state diagram of a digital system is shown in Fig.Q4(c). Obtain relevant state model.

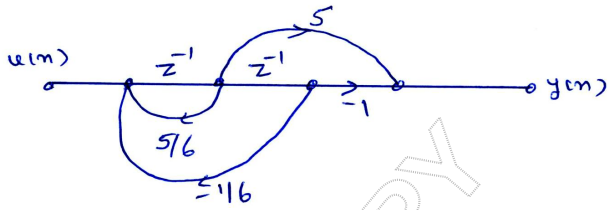

Fig.Q4(c) **(06 Marks)**

Module-3

5 a. For the state equation

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u ; \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Determine the state transition matrix. Also solve for the state vector x(t), when u(t) is a unit step function. **(10 Marks)**

b. Consider a continuous time system described by the state equation

$$\dot{x}(t) = A\ x(t) + B\ u(t)\ ;\ \ y = c\ x(t)\ ,$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $C = [\,1 \quad 0\,]$

Transform the state model to discrete form. Take $T_s = 1s$. **(10 Marks)**

OR

6 a. Given the system matrix A of state variable description of continuous time system.

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

Evaluate state transition matrix by (i) Cayley Hamilton theorem; ii) Diagonalisation **(10 Marks)**

b. A discrete time system is described by the state model.

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r(k)\ ;\ \ y(k) = [1 \quad 0]\, x(k) \qquad \text{Given } x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Determine i) Zero input response ii) Response of the system when $r(k) = \cos(k\pi)$. **(10 Marks)**

Module-4

7 a. With necessary state diagrams, illustrate the general concept of controllability and observability. **(06 Marks)**

b. State and prove the condition used to test controllability of a continuous linear time invariant system. **(06 Marks)**

c. Check for complete observability of the discrete time system given by

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -1 & -1 \end{bmatrix}; \quad G = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $x(k+1) = F\,x(k) + G\,u(k)$; $y(k) = C\,x(k) + D\,u(k)$

and $u(k) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $k \geq 0$; $y(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$; $y(1) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$; $y(2) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Hence determine x(0) from above information. **(08 Marks)**

OR

8 a. Investigate controllability and observability of following system.

$$x(k+1) = \begin{bmatrix} 1 & k \\ 0 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [\,1 \quad 1\,]\, x(k)$$ **(04 Marks)**

b. Describe controllability and observability test results applicable to state model represented in Jordon canonical form. **(04 Marks)**

c. Write necessary condition for a continuous time system which is controllable to remain controllable after discretization. **(06 Marks)**

d. Derive necessary transformation matrix to perform equivalent transformation of an controllable system with arbitrary system matrix A to controllability canonical form. **(06 Marks)**

Module-5

9 a. How controllability and observability affected by state feedback? **(04 Marks)**

b. Derive a transformation matrix P which transforms an uncontrollable state model with arbitrary A and B matrices to controllable canonical form. **(08 Marks)**

c. With necessary mathematical analysis describe the working of a deadbeat observer used with a discrete time system. **(08 Marks)**

OR

10 a. Transform the system with

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}; \qquad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; \qquad C = [\,1 \quad 0 \quad 0\,]$$

into equivalent observable companion form. **(06 Marks)**

b. For the system represented in Q10(a), derive a state feedback control law such that the closed loop poles are located at –1, –2, –3. **(08 Marks)**

c. With necessary block diagram, analytical expressions, illustrate the design of a full order observer **(06 Marks)**

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