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201BS006

Ph.D./M.S. (Engg.) by Research Degree Examination, December 2022
Integral Transforms and Calculus of Variations

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of

i) $\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^2$ ii) $\frac{\cos\sqrt{t}}{\sqrt{t}}$. **(10 Marks)**

b. Find the inverse Laplace transform $\frac{s}{(s^2+1)(s^2+4)(s^2+9)}$. **(10 Marks)**

OR

2 a. State and prove convolution theorem. **(10 Marks)**

b. Use transform method to solve $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$ with $x=2$ $\frac{dx}{dt}=-1$ at $t=0$. **(10 Marks)**

Module-2

3 a. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2}$, $m>0$.

(10 Marks)

b. Solve $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$ if $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $u(x, t)$ is bounded where $x > 0$, $t > 0$. **(10 Marks)**

OR

4 a. Find the Fourier sine and cosine transform of x^{n-1}, $n > 0$ and prove that $\frac{1}{\sqrt{x}}$ is self reciprocal under Fourier sine and cosine transforms. **(10 Marks)**

b. Using finite Fourier transform solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0$, $u(4, t) = 0$ and $u(x, 0) = 2x$ where $0 < x < 4$, $t > 0$. **(10 Marks)**

Module-3

5 a. State and prove Euler's equation. **(10 Marks)**

b. Solve the variational problem $\delta\int_1^2[x^2(y')^2+2y(x+y)]dx=0$ given $y(1) = y(2) = 0$.

(10 Marks)

OR

6 a. Define functional. Prove that the necessary condition for the functional $I = \int_{x_1}^{x_2} f(x, y, y')\,dx$ to be extremum is $\delta I = 0$. **(10 Marks)**

b. Find the extremal of the functional

$I = \int_0^{\pi/2} [y^2 - y'^2 - 2y \sin x]\,dx$ under the conditions $y(0) = y(\pi/2) = 0$. **(10 Marks)**

Module-4

7 a. State and solve the Brochistochrome problem. **(10 Marks)**

b. Find the extremal of the functional $I = \int_0^{\pi} (y'^2 - y^2)dx$ under the conditions $y(0) = 0$ $y(\pi) = 1$ and subject to the constraints $\int_0^{\pi} y\,dx = 1$. **(10 Marks)**

OR

8 a. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis gives a minimum surface area. **(10 Marks)**

b. Prove that the shortest distance between two points in a plane is a straight line. **(10 Marks)**

Module-5

9 a. State and prove Euler Poisson equation. **(10 Marks)**

b. Find the extremal of the functional $I = \int_0^1 (2\dot{x}^2 - 3\dot{y}^2 + x + y)\,dt$ given $x(0) = y(0) = 0$ $x(1) = y(1) = 1$. **(10 Marks)**

OR

10 a. Find the extremal of the functional $I = \int_{x1}^{x2} (y'''^2 + y^2 - 2yx^3)dx$. **(10 Marks)**

b. Obtain the extremal for the functional

$I = \int_0^{\pi/4} (\dot{x}^2 + \dot{y}^2 + 8xy)\,dt$ given $x(0) = 0$ $y(0) = 0$ $x(\pi/4) = 1$ $y(\pi/4) = 1$. **(10 Marks)**

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