CBCS SCHEME

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21MAT656

Sixth Semester B.E. Degree Examination, June/July 2024
Mathematics for Machine Learning

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Apply Gauss-Elimination method to solve the system of equations $2x + y + 4z = 12$, $4x + 11y - z = 33$ and $8x - 3y + 2z = 20$. **(06 Marks)**

b. Show that the vectors $v_1 = (1, 1, 2, 4)$; $v_2 = (2, -1, -5, 2)$; $v_3 = (1, -1, -4, 0)$ and $v_4 = (2, 1, 1, 6)$ are linearly dependent in $R^4(R)$. **(07 Marks)**

c. Define vector space. Write the polynomial $f(t) = at^2 + bt + c$ as a linear combination of the polynomial $p_1 = (t-1)^2$, $p_2 = t - 1$ and $p_3 = 1$. **(07 Marks)**

OR

2 a. Investigate for what values of λ and μ the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solution. **(06 Marks)**

b. Define an inner product space. Consider $f(t) = 3t - 5$ and $g(t) = t^2$ in the polynomial space $p(t)$ with inner product $\langle f.g \rangle = \int_0^1 f(t).g(t)dt$. Find $\langle f, g \rangle$, $\| f \|$ and $\| g \|$. **(07 Marks)**

c. Determine whether or not W is a subspace of $\mathbb{R}^3$ where $W = \{(a, b, c) \in R^3 : b = 3a, c = 2a\}$. Suppose the vector u, v, w are linearly independent. Show that the vectors $u + v$, $u - v$, $u - 2v + w$ are also linearly independent. **(07 Marks)**

Module-2

3 a. Let w be the subspace of $\mathbb{R}^5$ spanned by $u = (1, 2, 3 -1, 2)$ and $v = (2, 4, 7, 2, -1)$. Find the basis of the orthogonal complement $w^{\perp}$ of w. **(06 Marks)**

b. Let S consists of the following vectors in $\mathbb{R}^4$. $u_1 = (1, 1, 0, -1)$; $u_2 = (1, 2, 1, 3)$; $u_3 = (1, 1, -9, 2)$ and $u_4 = (16, -13, 1, 3)$
(i) Show that S is orthogonal and basis of $\mathbb{R}^4$.
(ii) Find the coordinates of an arbitrage vector $V = (a, b, c, d)$ relative to the basis S. **(07 Marks)**

c. Find (i) an orthogonal basis of W ; (ii) an orthogonal basis of W, where W is the subspace of $\mathbb{R}^4$ spanned by the vectors, $v_1 = (1, 1, 1, 1)$, $v_2 = (1, 1, 2, 4)$, $v_3 = (1, 2, -4, -3)$. **(07 Marks)**

OR

4 a. Find all the eigen values and the corresponding eigen vectors of the following matrix

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$ **(06 Marks)**

b. Find the matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$ **(07 Marks)**

Module-3

5 a. State the basic differentiation rules of univariate functions and using chain rule find the derivative of $(2x + 1)^4$. **(06 Marks)**

b. Define partial differentiation and gradients for the function $f(x, y) = x^2y + xy^3$, find the gradient. **(07 Marks)**

c. Compute the derivative of $f(x) = \exp\left(\frac{-1}{2\sigma^2}(x-\mu)^2\right)$, where μ and σ are constants. **(07 Marks)**

OR

6 a. Compute the derivate $f'(x)$ of the logistic sigmoid

$$f(x) = \frac{1}{1+\exp(-x)}$$ **(06 Marks)**

b. Differentiate f with respect to t and g with respect to X, where $f(t) = \sin(\log(f^T f))$, $t \in R^D$ $g(X) = \text{tra}(A\times B)$, $A \in R^{D\times E}$ $X = R^{E\times F}$, $B \in R^{F\times D}$. **(07 Marks)**

c. List of useful identities for computing Gradients. **(07 Marks)**

Module-4

7 a. For the following bivariate distribution P(x, y) of two discrete random variables X and Y.

$Y = y_1$	0.001	0.02	0.03	0.1	0.1
	x_1	x_2	x_3	x_4	x_5
		X			

Compute : The conditional distribution $P(x/Y = y_1)$ **(06 Marks)**

b. Bag1 contains four mangos and two apples; the Bag2 contains four mangos and four apples. A biased coin is tossed, which shows heads with probability 0.6 and tails with probability 0.4. If the coin shows heads we pick a fruit at random from Bag1 ; otherwise we pick a fruit at random from Bag2. A coin is tossed and a mango is picked from the bag. What is the probability that the mango was picked from Bag2? **(07 Marks)**

c. For the two random variable x, y and joint distribution P(x, y).
Show that $E_x[x] = E_Y[E_x[X/Y]]$, where $E_x[X/Y]$ denotes the expected value of x under the conditional distribution P(x/y). **(07 Marks)**

OR

8 a. Using a Bernoulli distribution with parameter μ : $P(X/\mu) = \mu^x(1-\mu)^{1-x}$, $x \in (0, 1)$. Choose a conjugate prior for the Bernoulli likelihood, compute the posterior distribution $P(\mu \mid x_1, x_2, \ldots x_N)$. **(06 Marks)**

b. Express the Bernoulli distribution in the natural form of the exponential family. **(07 Marks)**

c. For the mixture of two univariate Gaussian density $P(x) = \alpha p_1(x) + (1-\alpha)p_2(x)$, $0 < x < 1$, and $p_1(x)$ and $p_2(x)$ are univariate Gaussian densities with parameters, find the mean and variance of the mixture density p(x) **(07 Marks)**

Module-5

9 a. Find a stationary point and indicate whether they are maximum, minimum or a saddle points. **(06 Marks)**

b. Prove the following statements:
(i) The sum of any two convex function is convex.
(ii) The difference of any two convex functions is not a convex.
(iii) The product of any two convex functions is not convex function. **(07 Marks)**

c. Consider the following convex optimization problem:

$$\min_{w \in R^D} \frac{1}{2} W^T W \quad \text{subject to } W^T x \geq 1$$

Derive the Lagrangian dual by introducing the Lagrange multiplier λ. **(07 Marks)**

OR

10 a. Express the following optimization problem as a standard linear program in matrix notation.
$\max_{x \in R^2, \xi \in R} P^T x + \xi$, subject to the constraints that $\xi \geq 0$, $x_0 \leq 0$ and $x_1 \leq 3$. **(06 Marks)**

b. Consider the Quadratic program.

$$\min_{x \in R^2} \cdot \frac{1}{2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{Subject to } \begin{bmatrix} 41 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Derive the dual quadratic program using Lagrange duality. **(07 Marks)**

c. For the function $f(x) = \frac{1}{2} x^T A x + bTx + c$, A is strictly positive definite, it is invertible. Derive the convex conjugate of f(x). **(07 Marks)**

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